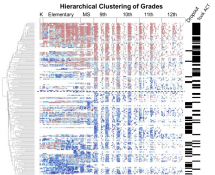
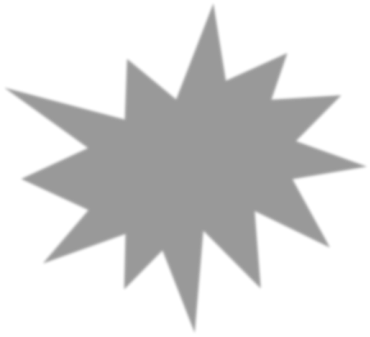
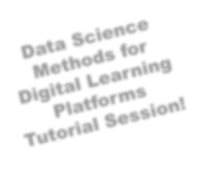
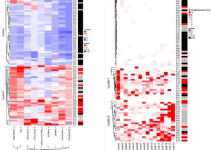
**Hierarchical Cluster Analysis Heatmaps in R Application and Methods for Education Research** 

Alex J. Bowers, Ph.D. 

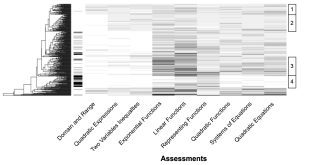
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Pattern Analysis with Cluster Analysis Heatmap Visualizations

1. Examples of hierarchical cluster analysis (HCA) heatmaps

• Non-cumulative grades K-12 to examine dropout

• Clickstream logfiles in Canvas to examine engagement and participation

• Number of times students retake Algebra I summative assessments

2. Cluster analysis vs. classification machine learning vs. inferential stats • Clustering (hierarchical, k-means, etc) vs. classifying (KNN, SVM, etc.)

• Unsupervised empirical clustering (hierarchical) vs making it up (k-means)

• There is no agreed method for the correct number of clusters (really?!? Yes!) • A brief tangent on Latent Class Analysis (LCA) vs. Cluster Analysis

3. Hierarchical Cluster Analysis Heatmaps

• Distance metrics (Euclidean and Uncentered Correlation)

• Agglomeration/clustering algorithm (average linkage)

• Heatmaps and cluster analysis - a research literature on visual data analysis 4. R Markdown example with “mtcars” dataset

• Data cleaning

• Plot the HCA heatmap

• Cluster using Euclidean distance, uncentered correlation, and average linkage • Add annotations for covariates and distal outcomes that are outside the clustering

Alex Bowers, 2024

Additional optional readings on cluster analysis in education

• Bowers, A.J. (2010) Analyzing the Longitudinal K-12 Grading Histories of Entire Cohorts of Students: Grades, Data Driven Decision Making, Dropping Out and Hierarchical Cluster Analysis. *Practical Assessment, Research & Evaluation* (PARE), 15(7), 1-18. https://doi.org/10.7275/r4zq-9c31

• Jorion, N., Roberts, J., Bowers, A.J., Tissenbaum, M., Lyons, L., Kuma, V., Berland, M. (2020) Uncovering Patterns in Constructionist Collaborative Learning Activities via Cluster Analysis of Museum Exhibit Log Files. *Frontline Learning Research*, 8(6), p.77-87. https://doi.org/10.14786/flr.v8i6.597

• Nitkin, D., Ready, D., Bowers, A.J. (2022) Utilizing Hierarchical Cluster Analysis and Heatmaps to Explore and Visualize Data Generated by a Technology-Based, Personalized Instructional Model. *Frontiers in Education*, 7:646471. https://doi.org/10.3389/feduc.2022.646471

• Lee, J., Recker, M., Bowers, A.J., Yuan, M. (2016). Hierarchical Cluster Analysis Heatmaps and Pattern Analysis: An Approach for Visualizing Learning Management System Interaction Data. Presented at the annual International Conference on Educational Data Mining (EDM), Raleigh, NC: June 2016. https://www.educationaldatamining.org/EDM2016/proceedings/paper\_34.pdf

• Reverter, A., Martinez, C., Currey, P., van Bommel, S., & Hudson, N. J. (2020). Unravelling student evaluations of courses and teachers. *Cogent Education*, 7(1), 1771830.

https://doi.org/10.1080/2331186X.2020.1771830

Alex Bowers, 2024

Persons

Variables

Cattell’s Data Box (1966)

Cattell, R. B. (1966). The data box: its ordering of total resources in terms of possible relational systems. 67-128. *Handbook of Multivariate Experimental Psychology*.

Boker, Steven (2017). Dynamical systems analysis in the context of statistical methods and research design. Keynote address, Modern Modeling Methods Conference, Storrs CT: May 22, 2017. Alex Bowers, 2024

Persons

Variables

Clinicians

Occasions x Variables

“diagnosis”

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Clinicians

Occasions x Variables

“diagnosis”

Persons

Persons

Variables

Variables

Educators

Persons x Occasions

“growth”

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Persons

Variables

Persons

Clinicians

Occasions x Variables “diagnosis”

Persons

Variables

Variables

Educators

Persons x Occasions

“growth”

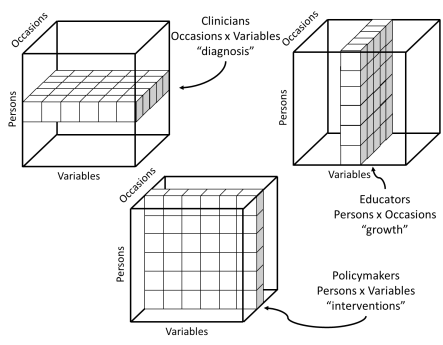
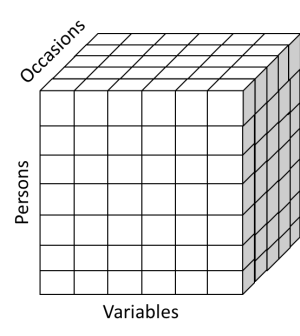
Policymakers

Persons x Variables

“interventions”

Alex Bowers, 2024

But how to describe the full Data Box all at once?



Cluster Analysis Heatmaps!

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What I Like about Cluster Analysis & Heatmap Visualizations Research Question possibilities:

• To what extent are there similar user data patterns?

• What do the data patterns look like?

• What does each data point for each individual look like patterned in context with all of the other data?

• How do the clusters relate to covariates and distal outcomes?

Method:

• Cluster analysis is a descriptive statistic

• No hypothesis test

• No agreed on method for the “correct” number of clusters.

• Few assumption violation issues and very robust to a wide range of data types • Attempts to use all of the available data and is robust to missing data issues • Heatmaps are adapted from bioinformatics and cancer biology

Alex Bowers, 2024

**Cluster Analysis Heatmap Definition**

*The cluster heat map is well known in the natural sciences and is one of the most widely used graphs in the biological sciences. As Weinstein (2008) notes,*

*"For visualization, by far the most popular graphical representation has been the clustered heat map, which compacts large amounts of information into a small space to bring out coherent patterns in the data.... Since their debut over 10 years ago, clustered heat maps have appeared in well over 4000 biological or biomedical publications.“*

Wilkinson & Friendly, 2009 (p.179)

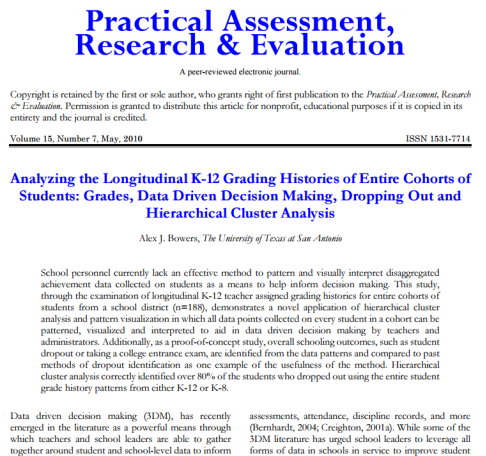
Weinstein, J. N. (2008). A Postgenomic Visual Icon. *Science*, 319(5871), 1772-1773.

https://doi.org/10.1126/science.1151888

Wilkinson, L., & Friendly, M. (2009). The History of the Cluster Heat Map. *The American Statistician*, 63(2), 179-184.

https://doi.org/10.1198/tas.2009.0033

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Bowers, A.J. (2010) Analyzing the Longitudinal K-12 Grading Histories of Entire Cohorts of Students: Grades, Data Driven Decision Making, Dropping Out and Hierarchical Cluster Analysis. *Practical Assessment, Research & Evaluation* (PARE), 15(7), 1-18. https://doi.org/10.7275/r4zq-9c31

Hypothetical Hierarchical Clustering Data

**Student 1**

**A**

**Student 2**

Grade Marking

**B**

**Student 3**

**Student 4**

**C**

**Student 5**

**D**

**Student 6**

**Student 7**

**F**

**Student 8**

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**K 1 2 3 4 5 6 7 8 9 10 11 12** Grade Year

Hypothetical Hierarchical Clustering Data

**Student 3 Student 7 Student 5 Student 4 Student 8 Student 6 Student 1 Student 2**

Grade Marking

**A**

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**B**

**C**

**D**

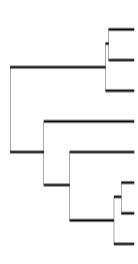
**F**

**K 1 2 3 4 5 6 7 8 9 10 11 12**

Grade Year

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Hypothetical Hierarchical Clustering Data

**Student 3 Student 7 Student 5 Student 4 Student 8 Student 6 Student 1 Student 2** 

Grade Marking

**A**

|  |  |  |  |  |  |  |  |  |  |  |
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**B**

**C**

**D**

**F**

**K 1 2 3 4 5 6 7 8 9 10 11 12**

Grade Year

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Hypothetical Hierarchical Clustering Data: Clustergram

**A**

|  |  |  |  |  |  |  |  |  |  |  |
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**B**

**C**

**D**

**F**

**K 1 2 3 4 5 6 7 8 9 10 11 12**

Grade Year

Alex Bowers, 2024

Student 3 Student 7 Student 5 Student 4 Student 8 Student 6 Student 1 Student 2

Hypothetical Hierarchical Clustering Data: Clustergram

**Grade Year**

K 1 2 3 4 5 6 7 8 9 10 1112

**A**

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**B**

**C**

**D**

**F**

**K 1 2 3 4 5 6 7 8 9 10 11 12**

Grade Year

Alex Bowers, 2024

Student 3

Student 7

Student 5

Student 4

Student 8

Student 6

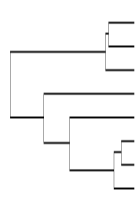
Student 1

Student 2

A B C D F

Hypothetical Hierarchical Clustering Data: Clustergram

**Grade Year**

K 1 2 3 4 5 6 7 8 9 10 1112 

Student 3

Student 7

Student 5

Student 4

Student 8

Student 6

Student 1

Student 2

A B C D F

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Hierarchical Clustering of Teacher Assigned Subject-Specific Grades K Elementary MS 9th 10th 11th 12th

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Hierarchical Clustering of Teacher Assigned Subject-Specific Grades K Elementary MS 9th 10th 11th 12th

Cluster Tree

Alex Bowers, 2024

Hierarchical Clustering of Teacher Assigned Subject-Specific Grades K Elementary MS 9th 10th 11th 12th

Cluster Tree

Alex Bowers, 2024

Hierarchical Clustering of Teacher Assigned Subject-Specific Grades

K Elementary MS 9th 10th 11th 12th

Subjects

Cluster Tree

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Hierarchical Clustering of Teacher Assigned Subject-Specific Grades

K Elementary MS 9th 10th 11th 12th

Subjects

Cluster Tree

Students

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Hierarchical Clustering of Teacher Assigned Subject-Specific Grades

K Elementary MS 9th 10th 11th 12th

Subjects

Cluster Tree

Students

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Hierarchical Clustering of Teacher Assigned Subject-Specific Grades

K Elementary MS 9th 10th 11th 12th

Subjects

Cluster Tree

Students

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Hierarchical Clustering of Teacher Assigned Subject-Specific Grades

K Elementary MS 9th 10th 11th 12th

Subjects

Cluster Tree

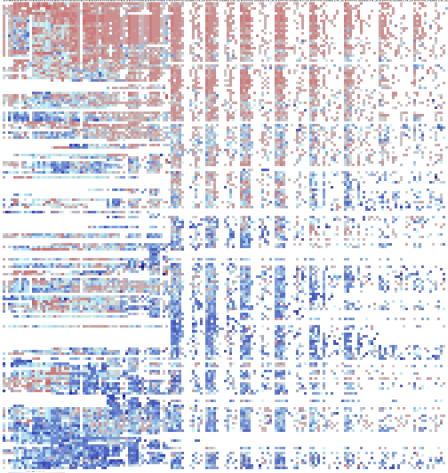
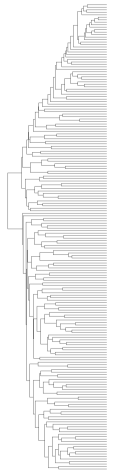
Students

+3 0 -3 No Data

Alex Bowers, 2024

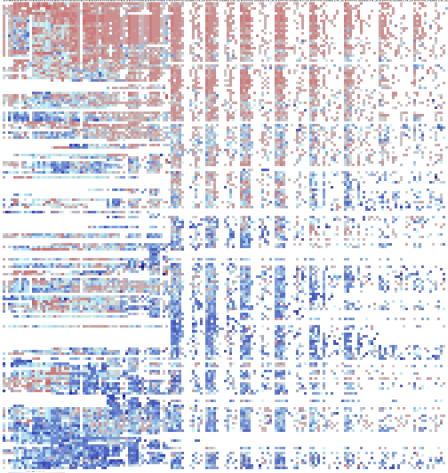
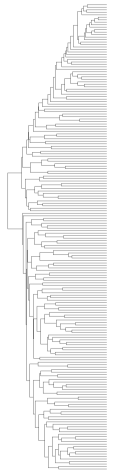
**Hierarchical Clustering of Grades**

K Elementary MS 9th 10th 11th 12th

Alex Bowers, 2024

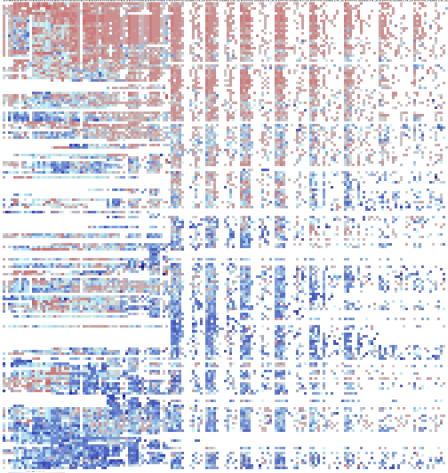
**Hierarchical Clustering of Grades**

K Elementary MS 9th 10th 11th 12th

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**Hierarchical Clustering of Grades**

K Elementary MS 9th 10th 11th 12th

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**Hierarchical Clustering of Grades**

K Elementary MS 9th 10th 11th 12th High-High

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**Hierarchical Clustering of Grades**

K Elementary MS 9th 10th 11th 12th High-High

Low-Low 

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**Hierarchical Clustering of Grades**

K Elementary MS 9th 10th 11th 12th High-High

Low-High 

Low-Low

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**Hierarchical Clustering of Grades**

K Elementary MS 9th 10th 11th 12th High-High

Low-High 

High-Low

Low-Low

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Mean non-cumulative GPA trends for clusters high-high, low-low, low-high and high-low, K-12

4

3.5

Mean Non-cumulative GPAHigh-High

3

2.5

2

1.5

1

0.5

0

K 1 2 3 4 5 6 7 8 9S1 9S2 10S1 10S2 11S1 11S2 12S1 12S2

Grade-Level

Alex Bowers, 2024

Mean non-cumulative GPA trends for clusters high-high, low-low, low-high and high-low, K-12

4

3.5

Mean Non-cumulative GPAHigh-High

3

2.5

2

1.5

1

Low-Low

0.5

0

K 1 2 3 4 5 6 7 8 9S1 9S2 10S1 10S2 11S1 11S2 12S1 12S2

Grade-Level

Alex Bowers, 2024

Mean non-cumulative GPA trends for clusters high-high, low-low, low-high and high-low, K-12

4

3.5

Mean Non-cumulative GPA

3

2.5

2

1.5

1

High-High

Low-Low

0.5

High-Low

0

K 1 2 3 4 5 6 7 8 9S1 9S2 10S1 10S2 11S1 11S2 12S1 12S2

Grade-Level

Alex Bowers, 2024

Mean non-cumulative GPA trends for clusters high-high, low-low, low-high and high-low, K-12

4

3.5

Mean Non-cumulative GPA

3

2.5

2

1.5

1

High-High

Low-Low

0.5

High-Low

Low-High

0

K 1 2 3 4 5 6 7 8 9S1 9S2 10S1 10S2 11S1 11S2 12S1 12S2

Grade-Level

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**Hierarchical Clustering of Grades**

K Elementary MS 9th 10th 11th 12th

Alex Bowers, 2024

**Hierarchical Cluster Analysis Heatmaps in R Application and Methods for Education Research** 

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http://www.tc.columbia.edu/elda/

ORCiD ID: 0000-0002-5140-6428

So what is a cluster analysis heatmap and what are the recommendations? Alex Bowers, 2024

**Cluster Analysis Heatmap Definition**

*For visualization, by far the most popular graphical representation has been the “clustered heat map,” which compacts large amounts of information into a small space to bring out coherent patterns in the data... In the case of gene expression data, the color assigned to a point in the heat map grid indicates how much of a particular RNA or protein is expressed in a given sample. The gene expression level is generally indicated by red for high expression and either green or blue for low expression. Coherent patterns (patches) of color are generated by hierarchical clustering on both horizontal and vertical axes to bring like together with like. Cluster relationships are indicated by tree-like structures adjacent to the heat map, and the patches of color may indicate functional relationships among genes and samples source of order other than clustering (for example, time in a series of measurements). -* Weinstein (2008) p.1772

Weinstein, J. N. (2008). A Postgenomic Visual Icon. *Science*, 319(5871), 1772-1773. https://doi.org/10.1126/science.1151888 Alex Bowers, 2024

Cluster Heat Maps have a Long History in Data Visualization Research 

Wilkinson, L., & Friendly, M. (2009). The History of the Cluster Heat Map. *The American Statistician*, 63(2), 179-184. https://doi.org/10.1198/tas.2009.0033

Alex Bowers, 2024

**Cluster Analysis Heatmap Limitations (quote from Weinstein, 2008)**

“*Seductive though it may be, the clustered heat map has its limitations and potential for misinterpretation or misuse:*

*1. Most prominently among the limitations, it provides only first order insight into the data; complex patterns of nonlinear relationship among only a few of the samples are unlikely to show up.*

*2. In hierarchical clustering, each bifurcation of the cluster tree can be “swung” in either direction at each fork in the tree, so some objective (but, to a degree, arbitrary) rule must be invoked to decide which way each branch will, in fact, swing.*

*3. There is also the temptation to select a small subset of the variables, and represent them in a clustered heat map.... However, if one picks a signature consisting of only a few dozen [features or variables] out of a set of more than 10,000, then even randomized data can produce clustered heat maps that appear spuriously to show good distinction of two subclasses.*

*Even beyond those limitations and concerns, the generation of clustered heat maps is a surprisingly subtle process…*” *-* Weinstein (2008) p.1772-1773

Weinstein, J. N. (2008). A Postgenomic Visual Icon. *Science*, 319(5871), 1772-1773. https://doi.org/10.1126/science.1151888 Alex Bowers, 2024

**Cluster Analysis Heatmap Limitations (quote from Weinstein, 2008)**

“*Seductive though it may be, the clustered heat map has its limitations and potential for misinterpretation or misuse:*

Cluster heatmaps are best for

*1. Most prominently among the limitations, it provides only first order insight into the data; complex*

comprehensive feature-rich

description.

*patterns of nonlinear relationship among only a few of the samples are unlikely to show up.*

*2. In hierarchical clustering, each bifurcation of the cluster tree can be “swung” in either direction at* Cluster heatmaps involve a long list of

*each fork in the tree, so some objective (but, to a degree, arbitrary) rule must be invoked to decide*

*which way each branch will, in fact, swing.*

arbitrary decisions. In your methods, report what you did.

*3. There is also the temptation to select a small subset of the variables, and represent them in a clustered heat map.... However, if one picks a signature consisting of only a few dozen [features or*

Cherry picking features, variables, and

*variables] out of a set of more than 10,000, then even randomized data can produce clustered heat*

individuals to include leads to spurious

*maps that appear spuriously to show good distinction of two subclasses.*

results.

*Even beyond those limitations and concerns, the generation of clustered heat maps is a surprisingly subtle process…*” *-* Weinstein (2008) p.1772-1773

Weinstein, J. N. (2008). A Postgenomic Visual Icon. *Science*, 319(5871), 1772-1773. https://doi.org/10.1126/science.1151888

Alex Bowers, 2024

**Cluster Analysis Heatmap Distance and Agglomeration Algorithms**

It might feel like this at the 

start, but you will get very

different outcomes depending

on the long list of choices…

Alex Bowers, 2024

Multiple types of clustering

• There are many many types of clustering. The top two are:

• Hierarchical (bottom-up – i.e. every row is a cluster and then agglomerate clusters using closest cluster) • K-means (top-down - iteratively identify k clusters by closest centroid. k is totally arbitrary, needs a random seed)

• Many others:

• Dynamic

• EM-algorithm distribution-based

• Spectral

• Density

• Graph-based

• And more!

• These are all descriptive statistics!

• There is no hypothesis test

• There is no agreed upon standard way on deciding how many clusters is the “right” number • Empirically cluster based on distance measure vs. machine learn/train on a type or profile • Very different from KNN (nearest neighbors) or other classifiers/recommenders

• Most clustering methods have big problems with missing data

• k-means has big problems with missing data

• Hierarchical clustering using average linkage is robust to missing data (takes the average) • Clustering has two major parts:

• Calculate a distance matrix

• Euclidean is usually default, but there are many others. I recommend “uncentered correlation” for education data. • Use an agglomeration algorithm

• I like average linkage (robust to missingness) but there many many others

Alex Bowers, 2024

https://twitter.com/jayelmnop/status/1258250020501032960

Alex Bowers, 2024



Romesburg, H. C. (1984). *Cluster analysis for researchers*. Belmont, CA: Lifetime Learning Publications. Alex Bowers, 2024

Xu, R., & Wunsch, D. (2005). Survey of Clustering Algorithms*. IEEE Trans On Neural Networks*, 16. https://doi.org/10.1109/tnn.2005.845141

My recommendations for cluster analysis heatmaps

1. Decide what are rows (students?) and what are columns (features?).

• Cluster just rows, just columns, or both?

• Time is a valid order for the columns. Bowers examples.

2. Decide what should be patterned, versus what are annotations.

• Covariates (gender, treatment, SES, classroom…)

• Distals (course completion, grade, graduation)

• Don’t put dropout into the pattern analysis to predict dropout!

3. Include all of the features and data that are relevant. This is descriptive, do not cherry pick. • From genomics, they include every gene they have access to (tens of thousands) by thousands of patient samples. The more, the better. 

4. Color map. Blue (low) to red (high). For colorblind people please!

5. Clustering algorithm

A

• Hierarchical vs. k-means

• Something else? Why? Look to bioinformatics literature if you’re going to “branch out”.

B

6. Distance metric

C

• Euclidean (shortest distance between two points in multidimensional feature space)

• Uncentered correlation (cosine angle)

D

7. Agglomeration method.

F

• Average linkage is robust to missing data and gives really good results.

*8. To tell the difference in the clusters I’ll just use PCA or t-SNE or something stats-y or deep learning and it’ll tell me which clusters are “real”.*

• Errrr….

• The bioinformatics literature on exactly this question is vast and remains an open question.

• Good reading before you go too far: Quackenbush, J. (2006). Microarray analysis and tumor classification. *The New England Journal of Medicine*, 354(23), 2463-2475.

• Use a cluster analysis heatmap to describe the data. It’s like a much better and comprehensive “table 1” for descriptive statistics on your features, because it shows all of the data and relationships for every individual. No averages.

?

Alex Bowers, 2024

An example using student grades 

with Euclidean distance and

Average Linkage



Alex Bowers, 2024

Alex Bowers, 2024



Average linkage, is the distance between

any two clusters A and B is defined as the

average distance of the total number of

cases within both clusters nAnB,

between the total number of cases in

cluster A, nA, and the total number of

cases in cluster B, nB



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Alex Bowers, 2024

Alex Bowers, 2024







Alex Bowers, 2024

Apply a Heatmap to the Clustered Data

Alex Bowers, 2024

**Hierarchical Cluster Analysis Heatmaps in R Application and Methods for Education Research** 

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Lee, J., Recker, M., Bowers, A.J., Yuan, M. (2016). Hierarchical Cluster Analysis Heatmaps and Pattern Analysis: An Approach for Visualizing Learning Management System Interaction Data. A poster presented at the annual International Conference on Educational Data Mining (EDM), Raleigh, NC: June 2016. http://www.educationaldatamining.org/EDM2016/proceedings/paper\_34.pdf Alex Bowers, 2024

• Canvas Learning Management System LMS Data

• Mid-sized University

• Undergrad freshman mathematics course, taught completely online as a required course • *n*=139 students 

• Clickstream LMS logfile data

• Features are # of pageviews

**Method:**

Hierarchical Cluster Analysis (HCA) Heatmaps

Clusters together students with similar longitudinal

data patterns and visualizes similarities and

differences Bowers (2007, 2010)

Alex Bowers, 2024

Cluster Analysis Heatmaps of Canvas Interaction Data

Pattern by Content 

Annotation column is final grade:

from red (high) to dark (low)

Student are rows

Course features

are columns

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Cluster Analysis Heatmaps of Canvas Interaction Data

Pattern by Content Pattern by Number of Interactions per Week 

Student are rows 

Course features

are columns

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Cluster Analysis Heatmaps of Canvas Interaction Data

Pattern by Content Pattern by Number of Interactions per Week

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Bowers, A.J., Krumm, A.E. (2021) Supporting Evidence-Based Improvement Cycles Through a Data-Intensive Partnership. *Information and Learning Sciences*, 112(9/10) 629-650. Journal Version: https://doi.org/10.1108/ILS-09-2020-0212 Open Access: https://doi.org/10.7916/d8-16m3-m804Alex Bowers, 2024

HCA Heatmap of Summative Assessment Attempts in Algebra I LMS **Figure 2**: *Hierarchical Cluster Analysis Heatmap of Student Algebra I Sub-Section Summative Assessments and Course Grades*.

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HCA Heatmap of Summative Assessment Attempts in Algebra I LMS Few attempts/high grade 

More attempts/medium grade

Few attempts/low grade

Course Grade

Dark is lower grade

Darker is more attempts

**Figure 2**: *Hierarchical Cluster Analysis Heatmap of Student Algebra I Sub-Section Summative Assessments and Course Grades*.

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HCA Heatmap of Summative Assessment Attempts in Algebra I LMS

Students retake Exponential Functions and

Linear Functions more often overall

Few attempts/high grade 

More attempts/medium grade

Few attempts/low grade

Course Grade

Dark is lower grade

Darker is more attempts

**Figure 2**: *Hierarchical Cluster Analysis Heatmap of Student Algebra I Sub-Section Summative Assessments and Course Grades*.

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*What is fascinating to me on this is a couple of thoughts: One general takeaway is not all assessments are created equal, even*

*though we treat them that way. It is so clear, if you had your year*

*broken into nine chunks of time [like in Figure 2] that the two in*

*the middle need more time [Exponential Functions and Linear*

*Functions], like the tests are harder… So we could smooth them*

*out or figure out how to represent that those weigh more. They*

*should have more weight on the way we think about helping a kid*

*go through this. –CMO leader*

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Ok, but if cluster analysis is really only descriptive, what do we do if we want to do subgroup analysis using inferential statistics?

Latent Class Analysis (LCA) is the answer

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Vermunt, J. K., & Magidson, J. (2002). Latent class cluster analysis. In J. A. Hagenaars & A. L. McCutcheon (Eds.), *Applied latent class analysis* (pp. 89-107): Cambridge University Press.Alex Bowers, 2024

*An important difference between standard cluster analysis techniques and LC clustering is that the latter is a model-based clustering approach. This means that a statistical model is postulated for the population from which the sample under study is taken. More precisely, it is assumed that the data are generated by a mixture of underlying probability distributions. When using the maximum-likelihood method for parameter estimation, the clustering problem involves maximizing a log-likelihood function. This is similar to standard nonhierarchical cluster techniques in which the allocation of objects to clusters should be optimal according to some criterion. These criteria typically involve minimizing the within cluster variation and/or maximizing the between-cluster variation. An advantage of using a statistical model is, however, that the choice of the cluster criterion is less arbitrary. –* Vermunt & Magidson (2002) p. 90

Vermunt, J. K., & Magidson, J. (2002). Latent class cluster analysis. In J. A. Hagenaars & A. L. McCutcheon (Eds.), *Applied latent class analysis* (pp. 89-107): Cambridge University Press.Alex Bowers, 2024

U1 U2 U3 U4 U5

Latent

XDistal

Class C

Outcome

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Current Recommendations for Correctly Specifying a 3-Step Latent Class Analysis

1. Nylund-Gibson, K., Grimm, R. P., & Masyn, K. E. (2019). Prediction from Latent Classes: A Demonstration of Different Approaches to Include Distal Outcomes in Mixture Models. *Structural Equation Modeling: A Multidisciplinary Journal*, 26(6), 967-985.

https://doi.org/10.1080/10705511.2019.1590146

2. Nylund-Gibson, K., & Choi, A. Y. (2018). Ten frequently asked questions about latent class analysis. *Translational Issues in Psychological Science*, 4(4), 440-461.

https://doi.org/10.1037/tps0000176

3. Collins, L. M., & Lanza, S. T. (2010). *Latent Class and Latent Transition Analysis: With Applications in the Social, Behavioral, and Health Sciences*. Hoboken, NJ: Wiley.

Example Application:

Duff, M., Bowers, A.J. (2022) Identifying a Typology of New York City Schools Through Teacher Perceptions of Organizational Capacity: A Latent Class Analysis. *Leadership and Policy in Schools*, 21(4), p.791-815. https://doi.org/10.1080/15700763.2020.1854789

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Back to cluster analysis heatmaps…

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Back to cluster analysis heatmaps…

Ok, great, but why should

we focus on a descriptive

method, even if it “has a

long history in

bioinformatics” and makes colorful figures?

So what? Who cares?

Where does this take 

learning analytics and

education research?

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https://twitter.com/DevilleSy/status/1305428369216016384

Take descriptive research more seriously

Traditionally, purely descriptive research—where researchers seek to characterize and explore relationships between measured variables without imputing causal explanations or testing elaborate verbal theories—is looked down on in many areas of psychology. This stigma discourages modesty, inhibits careful characterization of phenomena, and often leads to premature and overconfident efforts to assess simplistic theories that are hopelessly disconnected from the complexity of the real world (Cronbach, 1975; Rozin, 2001)…

We know that a large-scale shift in expectations regarding the utility of careful descriptive work is possible, because other fields have undergone such a transition to varying extents. Perhaps most notably, in statistical genetics, the small-sample candidate gene studies that made regular headlines in the 1990s (e.g., Ebstein et al., 1996; Lesch et al., 1996)—virtually all of which later turned out to be spurious (Chabris et al., 2012; Colhoun, McKeigue, & Davey Smith, 2003; Sullivan, 2007), and were motivated by elegant theoretical hypotheses that seem laughably simplistic in hindsight—have all but disappeared in favor of massive genome-wide association studies (GWAS) involving hundreds of thousands of subjects (Nagel et al., 2018; Savage et al., 2018; Wray et al., 2018). The latter are now considered the gold standard even in cases where they do little more than descriptively identify novel statistical associations between gene variants and behavior. In much of statistical genetics, at least, researchers seem to have accepted that the world is causally complicated, and attempting to obtain a reasonable descriptive characterization of some small part of it is a perfectly valid reason to conduct large, expensive empirical studies. - Yarkoni (2021) p.17

Yarkoni, T. (2021). The Generalizability Crisis. PsyArXiv. https://doi.org/10.31234/osf.io/jqw35

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Yarkoni, T. (2021). The Generalizability Crisis. PsyArXiv. https://doi.org/10.31234/osf.io/jqw35

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Lucas, C. et al., (2020). Longitudinal analyses reveal immunological misfiring in severe COVID-19. *Nature*, 584(7821), 463-469. https://doi.org/10.1038/s41586-020-2588-y Alex Bowers, 2024







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**Hierarchical Cluster Analysis Heatmaps in R Application and Methods for Education Research** 

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Towards Hierarchical Cluster Analysis Heatmaps as Visual Data Analysis of Entire Student Cohort Longitudinal Trajectories and Outcomes from Grade 9 through College

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36. https://doi.org/10.1353/hsj.2022.a906700

HCA Heatmap Shiny R web application: https://ohrice.shinyapps.io/Heatmap/ HCA Heatmap open access Shiny R application code: https://doi.org/10.7916/cqvn-9t71 HCA Heatmap open access R code tutorial: https://doi.org/10.7916/r1mg-yn37

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